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Localization and de-noising seismic signals on SASW measurement by wavelet transform



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ABSTRACT

SASW method is a nondestructive in situ testing method that is used to determine the dynamic properties of soil sites and pavement systems. Phase information and dispersion characteristics of a wave propagating through these systems have a significant role in the processing of recorded data. Inversion of the dispersive phase data provides information on the variation of shear-wave velocity with depth. However, in the case of sanded residual soil, it is not easy to produce the reliable phase spectrum curve. Due to natural noises and other human intervention in surface wave date generation deal with to reliable phase spectrum curve for sanded residual soil turn into the complex issue for geological scientist. In this paper, a time–frequency analysis based on complex Gaussian Derivative wavelet was applied to detect and localize all the events that are not identifiable by conventional signal processing methods. Then, the performance of discrete wavelet transform (DWT) in noise reduction of these recorded seismic signals was evaluated. Furthermore, in particular the influence of the decomposition level choice was investigated on efficiency of this process. This method is developed by various wavelet thresholding techniques which provide many options for controllable de-noising at each level of signal decomposition. Also, it obviates the need for high computation time compare with continuous wavelet transform. According to the results, the proposed method is powerful to visualize the interested spectrum range of seismic signals and to de-noise at low level decomposition.

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1. Introduction

Main results of geotechnical inquiries in recent years have proved that the dynamic shear modulus (G_{dyn}) is a basic stiffness at nondestructive strains (G_{max}) in static, cyclic and dynamic loading. The state parameter G_0 (e.g., Tatsuoka and Shibuya, 1991) is determined from the shear wave velocity (V_s) and total mass density (ρ_t) using either or both laboratory or in situ tests i.e. CHT, DHT, SCPT, SASW (Mayne et al., 2009).

$$G_0 = G_{\max} = G_{dyn} = \rho_t \cdot V_s^2 \tag{1}$$

The spectral analysis of surface waves (SASW) method is a nondestructive in situ testing technique that is used to determine dynamic soil and pavement properties, such as shear wave velocity V_S , shear modulus G, and damping ratio D (Kim and Park, 2002; Nazarian, 1984; Rosyidi and Taha, 2009a,b).

Instead of an accurate assessment importance of the dynamic parameters, in addition to the appropriate data acquisition, the

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E-mail addresses: golestani_alireza@civileng.iust.ac.ir (A. Golestani), mahdi_kolbadi@sina.kntu.ac.ir (S.M. S.Kolbadi), Heshmati@iust.ac.ir (A.A. Heshmati). data processing plays an unavoidable role in these determinations. In some investigations practically seismic tests, the recorded data and signal may be subjected to noises during the experimental process. Also, the recorded seismic signals are non-stationary due to their frequency content that varies in time. Therefore, a rigorous transform is needed to reveal time–frequency characteristics of the signals. Finally, numerous researchers are interestingly attentive to the signals that manipulate and select the appropriate method in signal post-processing.

These parameters can be determined based on dispersive characteristics of surface waves. Surface wave can propagate with different velocities in soil layers because its propagation velocity depends on wavelength (or frequency). For example, high-frequency waves (short wavelength) propagate only in near-surface layers and waves with longer wavelengths propagate through deeper layers as well as the near-surface layers (Kim and Park, 2002). As noted earlier, the required parameters are calculated as follow:

Phase information of the cross-power spectrum, which indicates the phase difference between two receiver signals or de-noised signals as a function of frequency, is obtainable. From the cross-power spectrum, time delay between receivers is calculated for each frequency by

$$t(f) = heta_{xy}(f)/2\pi f$$

(2)

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where $\theta_{xy}(f)$ is the phase shift of the cross-power spectrum in radian and the frequency, *f* is in cycles/s. The surface wave phase velocity, V_R is then determined using

$$V_R(f) = D/t(f) \tag{3}$$

where, *D* is the distance between two receivers. The corresponding wavelength of the surface wave, λ_{R} , is

$$\lambda_R(f) = V_R(f)/f. \tag{4}$$

These relations produced an experiment dispersion curve (λ_R versus V_R) for the receiver spacing (Kim and Park, 2002). Once all experimental dispersion curves for a group of receiver spacing are generated, they are combined together and a composite experimental dispersion curve is created. Therefore, in order to achieve reliable phase information while signals are disturbed by noises, it is necessary to implement a method to overcome the near-field effects.

It is generally accepted that modern wavelet transform analysis began in the early 1980s with the Morlet wavelet, produced to support the detailed investigation of seismic signals (E.g. Goupillaud et al., 1984). Many wavelets have been developed and utilized to analyze these and many other signals in geophysics.

Using wavelet transform in the time-frequency decomposition of seismic signals, it is possible to obtain accurate information of wave spectrum and to characterize the phase information of the transfer function spectrum. Decomposition of a seismic signal into a time-frequency format permits analysis and display of each frequency component in a unique and continuous format (Chik and Islam, 2009). The fast Fourier transform and the short time Fourier transform are two conventional methods for analysis of seismic signals. The Fourier analysis of seismic signals cannot show the local transient event due to averaging of signals. So, this may lead to some information lost in analyzing nonstationary signals (Chik and Islam, 2009; Rosyidi and Taha, 2009a,b). As the analysis window width is constant, the short time Fourier transform has a fixed time-frequency resolution. In order to pass through the existing limits, a new transform was highly required to allow time-frequency localization of the signals. The continuous wavelet transform (CWT) can overcome these drawbacks of conventional spectral transforms due to unlimited time-frequency resolution over the time-frequency space.

With a simple comparison between CWT and FFT ability it could be concluded that CWT can visualize a small discontinuity in the sinusoidal signal and it is not possible for FFT to show this tiny discontinuity. Wavelet transform's ability and high flexibility in signal processing and its application in various scientific fields have made it popular for most researchers in recent years. In the following, the role of wavelet transform is mentioned briefly in the review of recent development.

Rosyidi and Taha (2009a,b) conducted the study for the continuous wavelet transform on the signals recorded in the SASW test that was exposed in the environmental and background noises. They concluded that the CWT based on Gaussian derivative wavelet is a potential tool



Fig. 1. The SASW measurement setup (Kim and Park, 2002).

and spectral analysis of time-frequency decomposition for identification of transient events in non-stationary signal and filtering of noisy signals in seismic surface wave records. Kim and Park, 2002 utilized harmonic wavelet transform to determine dispersion curve of SASW test as an alternative to conventional phase unwrapping methods because the characteristic of Fourier transform used in this method is an erroneous dispersion curve. Finally they concluded that the new evaluation method is less affected by noise and near field effect (effect of body wave) than the phase unwrapping method. Park and Joh, 2009 applied the harmonic wavelet for phase spectrum assessment to remove background noise effects. They found that the proposed method successfully reduce noise effects and determine the phase spectrum more reliably than the conventional cross power spectrum under noisy field conditions. Park and Kim, 2000, proposed a new method using harmonic wavelet transform to evaluate dispersive phase and group velocities that could obtain these directly from information achieved from data based on the harmonic wavelet transform. Consequently, they confirmed the validity of the proposed method's results with the theoretical velocities in the multi-layered systems.

As mentioned above, several signal process methods based on wavelet transform have been developed and applied to remove noise effect. But, there are a few studies that examine the ability of wavelet transform in the various frequency events demonstration and de-noising, especially when determination of a trusted seismic spectrum response in soil layers aren't easily detectable due to environmental and background noises. In such cases, the frequency domain of noisy recorded signals is wide spread and the noises are in the lower and upper frequency ranges.

2. SASW method

The general configuration of accelerometers, source, and signal recording equipment in the SASW method is shown in Fig. 1. The two vertical accelerometers were placed on the paved ground surface at an equal distance from a fixed centerline. The source generated vertical displacement component was detected using these accelerometers. In the SASW measurement, only the vertical displacement component is of interest. Several configurations of the accelerometer and source spacings are required to measure the shear wave velocities of different depths (Rosyidi and Nayan, 2006; Rosyidi and Taha, 2009a,b). The applied measurement configuration in this study is the midpoint receiver spacing, i.e. the distance between the source and the near accelerometer set up equal to the distance between the accelerometers. The long accelerometer spacings of 80 and 160 cm with low frequency sources are used to measure the shear wave velocity of the subgrade layer. The SASW tests were carried out at several sites on the main road in the campus of the Universiti Kebangsaan Malaysia in Bangi, Selangor. All recorded signals are transformed using fast Fourier transform (FFT) to frequency domain. Previously, in the SASW method, the coherent signal averaging has been used to reduce the random noise level or eliminate incoherent signals but it does not always provide a clear guidance of frequency event of true surface wave signals (Rosyidi and Taha, 2009a,b). One of the main functions in the frequency domain between the two receivers is the phase information of the transfer function. The transfer function spectrum was applied to determine the relative phase shift between the two recorded signals in the range of the frequencies being generated (Nazarian, 1984). By unwrapping the phase information of transfer function, composite experimental curve was generated then the phase velocity was calculated using phase difference method. Eventually, the shear wave velocity of the soil layers was produced from an inversion process.

3. Wavelet transform and de-noising principles

The wavelet transform is a method of converting a function (or signal) into another form which either makes certain features of the



Fig. 2. 2 Levels DWT decomposition.

original signal more amenable to study or enables the original data set to be described more succinctly (Addison, 2002). The continuous wavelet transform is a rigorous method that was invented to overcome the limits of the fast and short time Fourier transform. To perform a wavelet transform, a localized waveform function that is called wavelet is needed. In order to be classified as a wavelet, a function must satisfy a certain mathematical criteria. A wavelet must have a finite energy (Addison, 2002; Stark, 2005):



Fig. 3. Flowchart of CWT and DWT systems.



Fig. 4. Seismic signals recorded from two accelerometers (channels 1 and 2).

where, E is the energy of a function equal to the integral of its squared magnitude. If $\psi(t)$ is a complex function the magnitude must be found using both its real and complex parts (Addison, 2002). If $\psi(f)$ is the Fourier transform of $\psi(t)$,

$$\psi(f) = \int_{-\infty}^{+\infty} \psi(t) e^{-i(2\pi f)t} dt \tag{6}$$

then the following condition must hold:

$$c_{g} = \int_{0}^{+\infty} \frac{\left|\psi(f)\right|^{2}}{f} df \,\langle \, \infty. \tag{7}$$

Eq. (3) is known as the admissibility condition and C_g is called the admissibility constant (Addison, 2002). Therefore, any finite energy function satisfying Eq. (3) will be called "wavelet" (Stark, 2005). An additional criterion that must be satisfied for complex wavelets is that the Fourier transform must both be real and vanish for negative frequencies. Wavelets show the signal properties in a more informative view. The shifted and dilated (or contracted) version of the original wavelet function is a more flexible wavelet function (Kaiser, 1994). This function is written as:

$$\psi_{a,b}(t) = \frac{1}{\sqrt{a}}\psi\left(\frac{(t-b)}{a}\right) \tag{8}$$

therefore, the CWT coefficients can be determined as following:

$$T(a,b) = \frac{1}{\sqrt{a}} \int_{-\infty}^{+\infty} x(t) \psi^*\left(\frac{(t-b)}{a}\right) dt$$
(9)

where T (a, b) are wavelet transform coefficients and the asterisk indicates that the complex conjugate of the wavelet function is applied in the transform. In this study, the complex Gaussian Derivative wavelet of order 32 is used to perform continuous wavelet transform. Generally, the complex Gaussian Derivative wavelet family is defined as following:

$$f(x) = C_p e^{-ix} e^{-x^2}$$
(10)

The integer P is the parameter of this family and C_p is such that $||f^{(P)}||^2 = 1$ where $f^{(P)}$ is the P^{th} derivative of f (Misiti and Misiti, 2008).

Calculating wavelet coefficients at every possible scale generates a lot of data. So in order to decrease the amount of work and number of wavelet transform coefficients, another method namely discrete wavelet transform (DWT) will be employed. DWT coefficients are usually sampled from the CWT on a dyadic grid (Matz and Kreidl, 2004). In this transform, scale and location parameters are defined as $a = 2^m$ and b = n. 2^m . Therefore, the normalized wavelet can be defined as following:

$$\psi_{m,n}(t) = \frac{1}{\sqrt{2^m}} \psi\left(\frac{(t-n2^m)}{2^m}\right).$$
 (11)

The main reason of employing discrete wavelet transform in this investigation is its high ability in decomposition and de-noising the recorded seismic signals. Based on the investigations done by Rosyidi et al., subgrade soil material of other sites tested in Kebangsaan University is sandy clayey residual soil and these materials are susceptible to random and coherent noise (Rosyidi and Taha, 2009a,b). Therefore, for determination of the seismic response spectrum of interested soil layer, the DWT performance



Fig. 5. Fourier amplitude and phase information of recorded seismic signals of channel 1 (a) and 2 (b).

in de-noising the signals is evaluated. DWT analyzes the signal by decomposing it into its coarse (approximation) and detail information, which is performed by using successive high-pass and low-pass filtering process (Matz and Kreidl, 2004).

$$y_{high}(k) = \sum_{n} S(n) \cdot g(2k - n)$$
(12)

$$y_{low}(k) = \sum_{n} S(n) \cdot h(2k-n)$$
(13)

where $y_{high}(k)$ and $y_{low}(k)$ are the outputs of the high-pass filters with impulse responses g and h, respectively, after subsampling by 2 for removal of even/odd samples (Matz and Kreidl, 2004; Misiti and Misiti, 2008). Decomposition process can be continued in next levels, decomposing the generated approximations of upper levels. For an example, a 2 level DWT decomposition can be represented in Fig. 2.

This paper uses the proposed method by Chik and Islam (2009) for decomposition and de-noising the surface waves signals recorded from SASW tests. In this technique, a proper wavelet form of the seismic signals is decomposed at the first and the next stages after soft thresholding of detailed coefficients, the main signal is reconstructed. The de-noising method that uses thresholding in wavelet domain has been introduced by Donoho (Donoho, 2002; El-Dahshan, 2010). In this method a modified version of universal thresholding rule where introduced that calculates threshold for each level separately as follow (Misiti and Misiti, 2008):

$$\lambda_j = \delta_j \sqrt{2 * \log N_j} \tag{14}$$

where N_j is the length of coefficients at *j*th level and δ_j is the standard deviation of noise. This noise reduction method works well for a wide class of signals. Signal reconstruction is performed using inverse discrete wavelet transform (IDWT). Selection of appropriate level decomposition is one of the main purposes of this study. The manipulation of recorded signals is performed according to the following flowchart that is illustrated in Fig. 3.

4. Results and discussion

4.1. Soil properties at tested sites

On the basis of the results provided by Rosyidi and Taha (2009a,b), soil material of the subgrade layer in SASW test sites is sandy soil. The average of inverted shear wave velocity for soil subgrade layer was found to be 178.69 m/s with a range of 116.44 to 263.23 m/s. The results show that soil classification using SASW measurement is reasonably in agreement with the soil properties obtained from laboratory tests.

4.2. Seismic response spectrum and continuous wavelet transform

Fig. 4 shows the recorded signals at accelerometer (channels 1 and 2). In this case, the accelerometer 1 and 2 were located at a distance of 160 and 320 cm from the impact source, respectively. The recorded signals are transient and non-stationary. From these signals, it can be found that higher amplitude is measured for the first mode of Rayleigh wave amplitude. Decreasing signal magnitude is recognized as the Rayleigh wave attenuation in the soil layer. Recorded signal at channel 2 was identified as a weak seismic wave and it can be recognize as an effect of environmental noise which may be generated from ground noise and human-made sources (Rosyidi and Taha, 2009a,b). As the



Fig. 6. Time-frequency spectrum of recorded signal at channel 1.

generated wave's energy attenuates with increasing the distance from source, some portion of amplitude attenuation is caused by increasing distance between the source and receiver 2.

Using the fast Fourier transform (FFT) analysis, both seismic signals were converted to Fourier amplitude and phase spectrum in the frequency domain (Fig. 5). Generally, three peaks of energy event with different frequency bands are observed in the Fourier spectrum of recorded signals. In both signals, the energy of the third event is lower than the other events.

The first event is recognized as the dominant low frequency peak which could be caused from high energy ground noise. The second event is the seismic signal of interest in this test. This frequency band is in good agreement with the seismic response spectrum (the results of CWT and coherence function in previous researches (Rosyidi and Taha, 2009a,b)) of soil subgrade layer at the other test sites of UKM's road. This event locates at a frequency band from 35 to 125 Hz (channel 1). However, in channel 2, the second event cannot be obviously determined in the spectrum due to low frequency noise event. The third event comes from the high frequency background noise which is known as the incoherent noise (Li and Tang, 2005). However, it is still hard to recognize the frequency band of concerned seismic signals.

The continuous wavelet transform using the complex Gaussian Derivative of order 32 is applied to accurate localization of noisy events from the seismic response spectrum of interested soil layer. In this new method, the wavelet coefficients are calculated in logarithmic form. It can well detect three various events and cover the broad range of the generated frequencies. The CWT spectrogram for recorded signals at channels 1 and 2 is shown in Figs. 6 and 7, respectively.

The CWT of Gaussian Derivative wavelet provides good resolution at different parts of the frequency band. From Figs. 6 and 7, three main energy events at different frequency bands were obviously localized. It can also be observed that low frequency energy event was found in the range of up to 25 Hz in both CWT spectrums and its significant energy is concentrated in the frequency level lower than 10 Hz. Of course, the energy of this frequency band of the second signal (channel 2) is more than the first signal and this results to a more difficult energy event detection using Fourier transform. This spectrum range is identified as environmental noise which was previously identified by Rosvidi and Taha (2009a,b). The second event is the interested seismic response spectrum. It was seen in the range of 35 to 130 Hz and 35 to 140 Hz for signals recorded at channels 1 and 2, respectively. The third event is in the range of 150 to 430 Hz which was identified as the effect of body wave propagation. From Fig. 7, the energy of the body waves decreases very guickly and only a small part of the frequency range of 150 to 430 Hz remains and this is due to increase of distance from the source and geometric damping effects (Wang and Siu, 2004). As seen in figures and, at high frequency range, the two time-frequency spectrum have no similarity due to existing random noise. The energy attenuation is obviously recognized from both CWT spectrograms.

4.3. Discrete wavelet transform and de-noising

For decomposition of the recorded signals it is very important to select a suitable wavelet. The shape of selected orthogonal wavelet has to be very similar to the seismic signals. Discrete wavelet decomposition provides space-saving coding, less time consuming (Mortazavi and Shahrtash, 2008) and is appropriate for exact reconstruction therefore; this method is utilized as different way with the



Fig. 7. Time-frequency spectrum of recorded signal at channel 2.



Fig. 8. Approximation of seismic signal of channel 1 at level 3.

previous research technique (Rosyidi and Taha, 2009a,b) to investigate the DWT performance in de-noising at each interested levels. A group of wavelets was tested: Daubechie's wavelet, the discrete Meyer wavelet and Coiflet's wavelet. The best results were gained



Fig. 9. Details of seismic signal of channel 1 at level 3.

by the discrete Meyer wavelet and using only this wavelet, the seismic signals can be reconstructed in the desired level decomposition without noise. In the following study, only the results of this wavelet are presented. For example, three-level decomposition of recorded seismic signals from channel 1 is shown in Figs. 8 and 9. The de-noising method that applies soft thresholding in wavelet domain is based on the proposed Donoho's method. After this stage, the inverse discrete wavelet transform (IDWT) returns a de-noised seismic signal at an interested decomposition level.

Fig. 10 demonstrates the application of the discrete Meyer wavelet in de-noising and reconstructing the recorded signals at 3 and 4 decomposition levels, respectively. Particularly for recorded signals on channel 2, the reconstructed signal improves the signal pattern of seismic surface waves.

The phase and Fourier spectrum from the de-noised signals are shown in Figs. 11 and 12. According to the phase and Fourier spectrum from de-noised seismic signals, de-noising at low level decomposition demonstrates better performance because at high level decomposition, the signal information is lost highly. The low level decomposition works well for high frequency noise eradication but it is not able to remove the low frequency noises (i.e. environmental noise). These results agree well with those found by Chik and Islam (2009). The limits and convenience of wavelet decomposition is revealed in this section which can assist for the choice of decomposition technique for geotechnical research.

5. Discussion

According to Figs. 6 and 7, the continuous wavelet transform technique revealed three frequency events with most energy spectrum detected as an intended event frequency of the soil layer. Logarithmic method is used to make feasible the calculation and visualization of wavelet coefficient of frequency spectrum due to a wide range of recorded signals. Also, the 3D presentation of the signals' events is a convenient way to find their exact location in a time–frequency spectrum.

Also, according to the results of discrete wavelet transform and de-noising, it detected that high frequency noises are well removed from signals and signal phase information are available more precisely. These results are found after applying various accessible DWT thresholding techniques. The best results are related to Donoho's method which is illustrated in this study. But, it is note-worthy that the desired signal in high-level decomposition loses some of the energy reduced in soft thresholding. Furthermore, in soft thresholding the wavelet coefficients are decreased by a quantity equal to the threshold value which will induce the deviation when the filtered signal is reconstructed (Yi et al., 2012). Thresholding wavelet coefficients and evaluation of its efficiency at each level of decomposition distinguish DWT de-noising from CWT because abundant produced coefficients of the latter method prevent from controllable de-noising.

6. Conclusion

The analytical application of wavelet transform indicates its ability to covering conventional spectral analysis disadvantages and provides time-frequency spectrum of seismic signals with more details. As demonstrated, continuous wavelet analysis using complex Gaussian derivative wavelet is able to distinguish the seismic response spectrum for concerning soil layer from other noisy parts of the signal and provides results which are in good agreement with previous researches. Using a new logarithmic method for calculating wavelet coefficients improved continuous wavelet transform that led to the revelation of the results of seismic signals in various frequency ranges. In addition, 3D visualization of CWT outputs stimulates the detection and localization of events. Its



Fig. 10. Noise reduction of recorded seismic signals.

results complete the process of decomposition levels' selection and therefore it can be obvious which level is more important to denoise with high sensitivity. Investigating discrete wavelet transform method using Meyer wavelet for signal decomposition to various resolution levels revealed its high ability and flexibility for de-noising signals. This enhanced method provides a visual



Fig. 11. Fourier amplitude and phase information of de-noised signal of channel 1 at levels 3 and 4.

controllable manipulation of decomposed signals at each level. The adjustable thresholding of DWT coefficients made it possible to select a convenient wavelet for de-noising. Soft thresholding of details' coefficients obtained using this approach led to an accurate eradication of high frequency noises. Also, increasing decomposition levels decreases the accuracy since it causes original signal



Fig. 12. Fourier amplitude and phase information of de-noised signal of channel 2 at levels 3 and 4.

distortion. This issue has been pointed out by previous researchers which imply its advantages for de-noising signals in low level decomposition.

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Appendix A. Supplementary data

Supplementary data to this article can be found online at http://dx. doi.org/10.1016/j.jappgeo.2013.08.010.

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